BIG O AND SMALL o NOTATIONS

SHANE CHERN

In analytic number theory, we often use the big O and small o notations, and their variants, when dealing with asymptotic analysis.

Bachmann–Landau notations: The big O notation[1] $f(x) = O(g(x))$ means that there exists an absolute constant C such that $|f(x)| \leq Cg(x)$. Also, the small o notation $f(x) = o(g(x))$ means that $\lim f(x)/g(x) = 0$.

Vinogradov notations: We say $f(x) \ll g(x)$ if $f(x) = O(g(x))$. Likewise, we say $f(x) \gg g(x)$ if $g(x) \ll f(x)$.

Asymptotic equivalence: We say $f(x) \sim g(x)$ if $\lim f(x)/g(x) = 1$.

Order of magnitude estimate: We say $f(x) \asymp g(x)$ if both $f(x) \ll g(x)$ and $g(x) \ll f(x)$ hold. Equivalently, there exist absolute constants C_1 and C_2 such that $C_1|g(x)| \leq |f(x)| \leq C_2|g(x)|$.

References

1. P. Bachmann, *Die Analytische Zahlentheorie* (in German), Leipzig, Teubner, 1894.

Department of Mathematics and Statistics, Dalhousie University, Halifax, Nova Scotia, B3H 4R2, Canada

E-mail address: chenxiaohang92@gmail.com

[1]Here O stands for "Ordnung", which means "order of" in German; see [1].